Null Singularity Formation by Scalar Fields in Colliding Waves and Black Holes

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Abstract

It was believed that when gravitational, electromagnetic and scalar waves interact, a *spacelike* curvature singularity or Cauchy horizon develops because of mutual focusing. We show with an **exact** solution that the collision of Einstein-Maxwell-Scalar fields, in contrast to previous studies, predicts singularities on *null* surfaces and that this is a transition phase between *spacelike* singularities and regular horizons. Divergences of tidal forces in the *null* singularities is shown to be weaker relative to the *spacelike* ones.

Using the local isometry between colliding plane waves and black holes, we show that the inner horizon of Reissner-Nordstrom black hole transforms into a *null* singularity when a particular scalar field is coupled to it. We also present an analytic **exact** solution, which represents a Reissner-Nordstrom black hole with scalar hair in between the ergosphere.

I. INTRODUCTION

Colliding plane waves (CPW) provide an excellent test bed toward a better understanding of singularities in general relativity. Khan and Penrose [1] considered the collision of two impulsive gravitational waves with parallel polarizations. They showed that a strong spacelike curvature singularity develops in the region of interaction. On the other hand, collision of gravitational waves with ones modified by coupling to sources can yield totally different results. We cite three examples of such cases.

First, the Bell-Szekeres (BS)[2] solution; this solution represents a collision of two constant profile electromagnetic (em) shock waves and its outcome is a quasiregular "singularity" in the interaction region. This is equivalent to a Cauchy- horizon (CH).

Second, the Chandrasekhar - Xanthopoulos (CX)[3] solution; this solution describes the collision of two impulsive gravitational waves accompanied by shock gravitational waves with non-parallel polarizations. It predicts the development of an event horizon. Analytic extension of the solution across the horizon reveals the existence of timelike singularities along two hyperbolic arcs that are locally isometric to the Kerr ergosphere region.

Third, again as shown by CX [4], coupling em waves to the solution given in ref. [3], develops a regular null hypersurface which is equivalent to a CH in the region of interaction. This region is locally isometric to the region of spacetime in between the two horizons named as event (outer) and Cauchy (inner) horizons of the Kerr-Newman (KN) black hole (BH).

The CH of BS solution was shown to be unstable against perturbations [5,6]. In ref. [7], it has been shown that, there is a similar inner horizon instability for BHs and the horizons change to spacelike singularities.

In brief, these examples conform to the earlier hypothesis that any horizon formed in CPW is null while any singularity formed is spacelike.

Later Ori [8] found that when the CH of a spinning BH is perturbed the result is a curvature singularity which has a null character rather than spacelike one. This new picture, compared to the previously accepted view attracted many researchers to confirm the same results. Burko [9,10], using numerical methods confirmed Ori's observation of a regular horizon changing to a null singularity when he applied a scalar field to a Reissner-Nordstrom (RN) BH.

The relations between the mathematical theory of BHs and of colliding waves, motivates us to explore analogous singularities in the space of colliding waves.

All analyses devoted to singularity formation in the context of CPWs result in non-null singularities except for the one considered by Ori in plane symmetric spacetimes [11]. Ori has discussed the null *weak* singularity in plane symmetric spacetimes, his arguments are generalized to CPW only by employing outgoing perturbation analyses. Although his for-

mulation for plane symmetric spacetime is justifiable, such an approach is insufficient to find an exact analytic solution to the Einstein field equations in the region of interaction. In this sense, the outcome of the outgoing perturbations does not reflect the real physical situation. The physical reality reveals the metric component $e^{-U(u,v)} = 0$ on the null hypersurface and causes a degeneracy in the metric. This degeneracy plays a crucial role on the *weak* or *strong* character of the singularity.

In this paper we close this gap by constructing an example of colliding Einstein-Maxwell-Scalar (EMS) waves which leads to a null singularity. In our example of colliding EMS waves leads us to another crucial point. Pure em shock waves with constant amplitudes yield regular horizons (see BS solution cited above). On the other hand collision of pure scalar waves as we show here yields spacelike singularities. We show that by coupling suitable em fields to the scalar field these space-like singularities are transformed into null singularities, suggesting that null singularities are intermediate formations (in other words a transition phase) between horizons and spacelike singularities. We also show that tidal accelerations at the null singularities have weaker divergences than in the case of spacelike singularities and naturally this raises the following question: Is it possible to manipulate appropriate counter plane waves so that we can completely eliminate the divergences ?. Although the answer to this question has so far been negative our example at least verifies that counter- em terms can be employed to weaken the singularity.

We note that in conformally flat space-times occurence of null singularities makes some tidal forces to be finite [12]. Our case here is not conformally flat and all our tidal forces turn out to be divergent, albeit weaker than before.

The paper is organized as follows. In section II we give the metric for a new class of colliding EMS and ES fields and their extension to the incoming regions. In section III we discuss the singularity structure by analysing the geodesic behaviour and tidal accelerations. In section IV we couple a scalar field to a linearly polarized version of CX [4] metric that results in null singularities in the non-spherical extension of the Reissner-Nordstrom (RN) BH spacetime. We also show a particular scalar hair confined in the ergosphere of RN BH that does not violate the BH property. The paper is concluded with a discussion in section V.

II. A NEW CPW GEOMETRY WITH A NULL SINGULARITY

A long time ago Penney gave a solution for EMS fields in spherical symmetry generalizing the RN solution with the addition of a scalar field [13]. In a similar manner by replacing spherical symmetry with planar symmetry and introducing the BS solution instead of RN we obtain a new solution in the theory of CPWs. We do this by checking all separate Maxwell, scalar and EMS field equations with appropriate boundary conditions. These include continuity of metric components with sourceless scalar and em field equations satisfied at the boundaries. Some first and second derivatives, however, contain discontinuities (or jumps) as was discussed first by O'Brien and Synge [14]. Later on BS and CX gave explicit examples of solutions within the context of Einstein-Maxwell theory of CPW that justified

the discontinuities in some of the energy-momentum components.
The inclusion of a scalar field, as we advocate here, is not an exception to the reality of discontinuities while we cross from the incoming region to the region of interaction.
Our line element describing the collision of linearly polarized EMS fields is summarized by,

$$ds^2 = \Delta^{1-A} Z^2 \left(\frac{d\tau^2}{\Delta} - \frac{d\sigma^2}{\delta} - \delta dx^2 \right) - \Delta^A Z^{-2} dy^2 \tag{1}$$

where the notation used is,

$$\begin{aligned} \tau + \sigma &= 2P\sqrt{1 - Q^2} \\ \tau - \sigma &= 2Q\sqrt{1 - P^2} \\ \Delta &= 1 - \tau^2 \\ \delta &= 1 - \sigma^2 \\ 2Z &= a(1 + \tau)^A + b(1 - \tau)^A \end{aligned} \tag{2}$$

with $P = u\theta(u)$ and $Q = v\theta(v)$, in which (u, v) are null coordinates and $\theta(x)$ stands for the step function. We choose the constant A, $0 < A < 1$ to represent a scalar parameter so that a scalar charge can be defined by $\sqrt{1 - A^2}$. Namely, for $A = 0$ we have the maximum scalar charge of unity, while for $A = 1$ the scalar charge vanishes. The constants (a, b) stand for two additional parameters with $a > 0$ and $b > 0$. The fact that metric (1) describes colliding EMS fields will be justified in the sequel. As particular limits, of (1) we observe the following.

i) For $A = 1$ (and $a = b$), it reduces to the well known BS solution of colliding constant profile em shock waves. This particular solution is known to possess a horizon in the interaction region.

ii) For $A = 0$, which implies a maximum scalar charge of unity it represents a collision of Einstein-Scalar (ES) fields that create a spacelike scalar curvature singularity. This will be discussed separately in section 2.2.

Our main concern in this paper is to investigate the effect of a scalar field on the formation of a null singularity. The massless scalar field and Maxwell equations

$$\partial_\mu \left(\sqrt{-g} g^{\mu\nu} \phi_\nu \right) = 0 \tag{3}$$

$$\partial_\mu \left(\sqrt{-g} F^{\mu\nu} \right) = 0 \tag{4}$$

are both satisfied by the scalar field

$$\phi(\tau) = \frac{1}{2} \sqrt{1 - A^2} \ln \left| \frac{1 + \tau}{1 - \tau} \right| \tag{5}$$

and the em vector potential

$$A_\mu = 2\delta_\mu^x \sqrt{ab} A\sigma, \tag{6}$$

respectively. This implies that for $A = 0$ there exists only a background scalar field and the solution given in metric (1) represents the collision of ES fields. A scalar curvature singularity forms in the region of interaction and has a spacelike character. As we increase A toward unity the scalar field diminishes and the singularity of the spacetime transforms to a Cauchy horizon. In the interval $0 < A < 1$, we have the case of a null singularity.
In Appendix A, we have shown that the Weyl and curvature scalars diverge as $\tau \to 1$ and this is interpreted as a scalar curvature singularity. However, the fundamental property is that *the present solution does not become singular on any spacelike surface in the region of interaction*. This can be seen as follows.
We define the singular surface as

$$S(\tau) = 1 - \tau \tag{7}$$

The normal vector to this surface is

$$(\nabla S)^2 = g^{\mu\nu} S_\mu S_\nu = g^{\tau\tau} S_\tau^2 = \Delta^A Z^{-2} = (1-\tau^2)^A Z^{-2} \tag{8}$$

As $\tau \to 1$ then $(\nabla S)^2 \to 0$ which indicates a null character orthogonal to both of the null directions of the incoming regions. It is also interesting to check that the line element (1) becomes null (i.e. $ds^2 = 0$) as $\tau \to 1$ for $0 < A < 1$ and $0 < u, v < 1$. This type of singularity is the first of its kind encountered in CPWs. In the obtained solution this null singularity emerges as an intermediate stage between the regular horizon and a space-like singularity.
To make this point more clear we employ the following successive transformations. First we rewrite the metric (1) in terms of new variables ω and r defined by;

$$\omega = \sqrt{\Delta\delta} = 1 - u^2 - v^2 \tag{9}$$

$$r = \tau\sigma = u^2 - v^2 \tag{10}$$

and metric (1) becomes

$$ds^2 = \Delta^{1-A} Z^2 \left(\frac{d\omega^2 - dr^2}{\tau^2 - \sigma^2} - \delta dx^2 \right) - \Delta^A Z^{-2} dy^2 \tag{11}$$

inverting the transformations (9) and (10) leads,

$$2\sigma^2 = 1 + r^2 - \omega^2 - \sqrt{D} \tag{12}$$

$$2\tau^2 = 1 + r^2 - \omega^2 + \sqrt{D} \tag{13}$$

where $D = (1 + r^2 - \omega^2)^2 - 4r^2 \geq 0$.
Secondly we set;

$$\begin{aligned}\omega + r &= \xi \\ \omega - r &= \eta\end{aligned} \tag{14}$$

Such that the equations (12) and (13) become,

$$2\sigma^2 = 1 - \xi\eta - \sqrt{(1-\xi^2)(1-\eta^2)} \tag{15}$$

$$2\tau^2 = 1 - \xi\eta + \sqrt{(1-\xi^2)(1-\eta^2)} \tag{16}$$

and the metric (11) takes the following form,

$$\begin{aligned} ds^2 = &\left[\frac{1}{2}(1+\xi\eta - \sqrt{(1-\xi^2)(1-\eta^2)})\right]^{1-A} F^2(\xi,\eta)\left[\frac{d\xi d\eta}{\sqrt{(1-\xi^2)(1-\eta^2)}}\right. \\ &\left. - \frac{1}{2}(1+\xi\eta + \sqrt{(1-\xi^2)(1-\eta^2)})dx^2\right] \\ &- \left[\frac{1}{2}(1+\xi\eta - \sqrt{(1-\xi^2)(1-\eta^2)})\right]^{A} F^{-2}(\xi,\eta)dy^2 \end{aligned} \tag{17}$$

where $F(\xi,\eta) = \frac{1}{2}Z(\xi,\eta)$.

The corresponding spacetime manifolds with coordinates (u, v, x, y) and (ξ, η, x, y) are illustrated in Fig.'s 1 and 2 respectively. It should be noted that, the description of the spacetime in the (ξ, η) coordinates breaks down on the null boundaries separating the interaction region (region IV) from the incoming regions (regions II and III), when $u = 0$ and $0 \leq v \leq 1$ or $v = 0$ and $0 \leq u \leq 1$. These points correspond to $\xi = 1$ or $\eta = 1$ respectively and the quantity $\sqrt{(1-\xi^2)(1-\eta^2)}$ in the metric (17) becomes zero. To avoid this problem the new variables ξ and η are restricted by the following inequality.

$$0 \leq \xi, \eta < 1 \tag{18}$$

Physically this means that metric (17) represents the interaction region only, and hence the null boundaries denoted by the points A and B in Figure 1. are excluded.
The metric (1) has another interesting property as far as spherical symmetry is concerned. By taking Z as

$$2Z = a_0|1+\tau|^A - b_0|1-\tau|^A \tag{19}$$

and using the transformations

$$\tau = \frac{m-r}{\sqrt{m^2-Q^2}}, \quad x = \phi, \quad y = (\sqrt{m^2-Q^2})t, \quad \sigma = \cos\theta \tag{20}$$

with $Q^2 = \frac{e^2}{A^2}$, where e is an electric charge, transforms metric (1) into

$$ds^2 = e^{-\alpha}dt^2 - e^{\alpha}dr^2 - e^{\beta}d\Omega^2 \tag{21}$$

Here we have

$$e^{\alpha} = [(r-a_0)(r-b_0)]^{-A}\left\{\frac{b_0|r-a_0|^A - a_0|r-b_0|^A}{b_0-a_0}\right\}^2$$

$$e^{\beta} = [(r-a_0)(r-b_0)]e^{\alpha}$$

$$a_0 = m - \sqrt{m^2 - \frac{e^2}{A^2}}$$

$$b_0 = m + \sqrt{m^2 - \frac{e^2}{A^2}} \tag{22}$$

Metric (21) is recognized as the solution by Penney [13], representing the generalization of RN solution in the presence of a massless scalar field. For $A = 1$ the solution reduces to RN BH.

A. Extension of the Space-Time into the Incoming Regions

The metric in region IV (i.e. the interaction region for $u > 0, v > 0$) can be extended across the null boundaries to find the incoming waves that participate in the collision. For example region II ($u > 0, v < 0$) is one of the incoming regions and the metric in this region is given by

$$ds^2 = 4(1-u^2)^{\frac{1}{2}-A}Z^2 du dv - (1-u^2)\left[\frac{Z^2}{(1-u^2)^{A-1}}dx^2 + \frac{(1-u^2)^{A-1}}{Z^2}dy^2\right] \tag{23}$$

where $2Z = a(1+u)^A + b(1-u)^A$. The non-zero scale invariant Weyl and Ricci scalars in this region are obtained from those of Appendix A (by imposing $v < 0$) as

$$\begin{aligned}
\Psi_4^{(0)} = & -\frac{A}{2}\left(\frac{a-b}{a+b}\right)\delta(u) - \frac{\theta(u)}{1-u^2}\left\{\frac{(2A-1)\left[2u^2(1-A)+1\right]}{1-u^2}\right. \\
& + \frac{A(A-1)}{Z}\left[a(1-3u)(1+u)^{A-1} + b(1+3u)(1-u)^{A-1}\right] \\
& - \frac{3A^2(1-u^2)}{2Z^2}\left[a(1+u)^{A-1} - b(1-u)^{A-1}\right]^2 - \frac{1}{1-u^2}\left[1+2u^2(A-1)\right] \\
& \left. + \frac{A\left[a(1+u)^{A-1} - b(1-u)^{A-1}\right]}{Z}\right\}
\end{aligned} \tag{24}$$

$$\begin{aligned}
\Phi_{22}^{(0)} = & \frac{\theta(u)}{4Z^2(1-u^2)^2}\left\{(1-A^2)\left[b^2(1-u)^{2A} + a^2(1+u)^{2A}\right]\right. \\
& \left. + 2ab(1+A^2)(1-u^2)^A\right\}
\end{aligned} \tag{25}$$

The incoming scalar field and the em vector potential are given by

$$\phi(u) = \frac{1}{2}\sqrt{1-A^2}\ln\left|\frac{1+u}{1-u}\right|, \tag{26}$$

and

$$A_\mu(u) = 2\sqrt{ab}\delta^x_\mu u, \tag{27}$$

respectively.

It is observed that an impulsive gravitational wave component (i.e. $\delta(u)$ term), arises only for $a \neq b$ and $A \neq 0$. For $A = 0$ both the impulsive term and the em field drops out leaving only a scalar field and therefore a colliding ES system. It is also clear that an impulsive term does not exist in the source $\Phi^{(0)}_{22}$ which implies the absence of a null shell. The positive definiteness of the incoming total energy of our combined em and scalar fields is crucial and obviously holds true.

The nature of the singularity in the incoming region is investigated by calculating the Riemann tensors both in local and PPON frames. In local coordinates the non-zero components are

$$\begin{aligned}
-R_{uxux} &= e^{V-U}\left[\Phi^{(0)}_{22} + \Psi^{(0)}_4\right] \\
-R_{uyuy} &= e^{-V-U}\left[\Phi^{(0)}_{22} - \Psi^{(0)}_4\right]
\end{aligned} \tag{28}$$

To find the Riemann tensor in a PPON frame, we define the following PPON frame vectors

$$\begin{aligned}
e^\mu_{(0)} &= \left(\frac{1}{2F}, \frac{1}{2}, 0, 0\right) \\
e^\mu_{(1)} &= \left(\frac{1}{2F}, -\frac{1}{2}, 0, 0\right) \\
e^\mu_{(2)} &= \left(0, 0, -e^{\frac{U-V}{2}}, 0\right) \\
e^\mu_{(3)} &= \left(0, 0, 0, -e^{\frac{U+V}{2}}\right)
\end{aligned} \tag{29}$$

Non-zero components in PPON frame that represent the tidal force components are

$$\begin{aligned}
R_{0202} = R_{0212} = R_{1212} &= -\frac{1}{4F^2}\left[\Phi^{(0)}_{22} + \Psi^{(0)}_4\right] \\
R_{0303} = R_{0313} = R_{1313} &= -\frac{1}{4F^2}\left[\Phi^{(0)}_{22} - \Psi^{(0)}_4\right]
\end{aligned} \tag{30}$$

where

$$\begin{aligned}
e^V &= (1-u^2)^{1-A}Z^2 \\
e^{-U} &= 1-u^2 \\
F &= (1-u^2)^{1/2-A}Z^2
\end{aligned} \tag{31}$$

It is clear to see that as $u \to 1$ all of these components diverge indicating a coordinate singularity. This is a non-scalar curvature singularity since all scalars in the incoming region

trivially vanish. Note that as $A \to 1$ the rate of divergence slows down and when $A = 1$ all the Riemann components become finite.
We also study the geodesics behaviour near the null singular surface. We choose the Lagrangian $L = (ds/d\lambda)^2$, where ds^2 is the line element in equation (1) and λ is an affine parameter. In addition to the energy ϵ we have three conserved momenta. These are

$$P_x = -e^{V-U}\dot{x} \tag{32}$$
$$P_y = -e^{-V-U}\dot{y} \tag{33}$$
$$P_v = e^{-M}\dot{u} \tag{34}$$
$$\epsilon = 2P_v\dot{v} - P_x^2 e^{U-V} - P_y^2 e^{U+V} \tag{35}$$

where ϵ may be taken 0 for null geodesics and 1 for timelike geodesics and dot represents derivative with respect to an affine parameter. Using equation (34) and (35) we obtain the following equation,

$$2P_v^2 e^M \frac{dv}{du} = \epsilon + P_x^2 e^{U-V} + P_y^2 e^{U+V} \tag{36}$$

The geodesic that remains in region II is obtained for $P_x = 0$. Integrating the above equation yields

$$v - v_0 = \frac{1}{2P_v^2}\left(\epsilon I_1 + P_y^2 I_2\right) \tag{37}$$

where $v_0 < -\frac{1}{2P_v^2}\left(\epsilon I_1 + P_y^2 I_2\right)$, with

$$\begin{aligned}
I_1 &= 2a^2 B_{\frac{1}{2}}\left[\frac{3}{2} - A, \frac{3}{2} + A\right] + 2b^2 B_{\frac{1}{2}}\left[\frac{3}{2} + A, \frac{3}{2} - A\right] + \frac{ab\pi}{4} \\
I_2 &= \frac{a^4}{2} B_{\frac{1}{2}}\left[\frac{3}{2} - 2A, \frac{3}{2} + 2A\right] + \frac{b^4}{2} B_{\frac{1}{2}}\left[\frac{3}{2} + 2A, \frac{3}{2} - 2A\right] \\
&\quad + 2a^3 b B_{\frac{1}{2}}\left[\frac{3}{2} - A, \frac{3}{2} + A\right] + 2ab^3 B_{\frac{1}{2}}\left[\frac{3}{2} + A, \frac{3}{2} - A\right] \\
&\quad + \frac{6a^2b^2\pi}{32}
\end{aligned}$$

for $0 < A < \frac{3}{4}$. Our notation $B_\lambda[\mu, \nu]$ represents the incomplete beta function which is defined in terms of the hypergeometric function by

$$\begin{aligned}
B_\lambda[\mu, \nu] &= \int_0^\lambda t^{\mu-1}(1-t)^{\nu-1}dt = \mu^{-1}\lambda^\mu F(\mu, 1-\nu; \mu+1; \lambda) \\
& 0 \le \lambda \le 1 \\
& \mu, \nu > 0
\end{aligned} \tag{38}$$

For $P_x \neq 0$ and $u < 1, v$ becomes positive and indicates that particles starting from $u = 0$ in region II can pass to region IV and they hit the scalar curvature singularity. Null geodesics of region II terminate their trajectories in the null singularity of the same region. In this manner the null singular surface does not change the general behaviour of particles motion in region II. In summary the geodesics behaviour in the present case is exactly similar to those considered by Matzner and Tipler [15] for the case of Khan-Penrose and the BS solutions.

B. The A=0 case and colliding ES waves

In this section we show explicitly that $A = 0$ in metric (1) describes colliding ES waves. The generic form of the colliding waves with linear polarization is described by the line element

$$ds^2 = 2e^{-M}dudv - e^{-U}\left(e^V dx^2 + e^{-V}dy^2\right) \tag{39}$$

and the field equations are as follows [16,17].

$$\begin{aligned} U_{uv} &= U_u U_v - 2(\Phi_{11}^{(0)} + 3\Lambda^{(0)}) && (40)\\ 2U_{uu} &= U_u^2 + V_u^2 - 2U_u M_u + 4\Phi_{22}^{(0)} && (41)\\ 2U_{vv} &= U_v^2 + V_v^2 - 2U_v M_v + 4\Phi_{00}^{(0)} && (42)\\ 2M_{uv} &= -U_u U_v + V_u V_v + 8\Phi_{11}^{(0)} && (43)\\ 4\Phi_{02}^{(0)} &= 4\Phi_{20}^{(0)} = 2V_{uv} - U_u V_v - U_v V_u && (44)\\ 2\phi_{uv} &= U_u \phi_v + U_v \phi_u && (45) \end{aligned}$$

The solution follows from Eq. (1) upon substitution of $A = 0$ which yields

$$\begin{aligned} e^{-U} &= 1 - u^2 - v^2 = \sqrt{\Delta\delta}\\ e^{-M} &= \frac{2\Delta Z_0^2}{\sqrt{1-u^2}\sqrt{1-v^2}}\\ e^V &= Z_0^2 e^{-U}\\ \phi &= \frac{1}{2}\ln\left|\frac{1+\tau}{1-\tau}\right| \end{aligned} \tag{46}$$

where $2Z_0 = a + b = const.$ and the null coordinates (u, v) are to be considered with the step functions $(\theta(u), \theta(v))$ respectively.
The non-zero Weyl and Ricci tetrad scalars follow from the Appendix A by setting $A = 0$,

$$2\Phi_{11}^{(0)} = 3\Psi_2^{(0)} = -6\Lambda^{(0)} = \phi_u\phi_v = \frac{\theta(u)\theta(v)}{\sqrt{1-u^2}\sqrt{1-v^2}\Delta} \tag{47}$$

$$\Phi_{22}^{(0)} = \Psi_4^{(0)} = \phi_u^2 = \frac{\theta(u)}{(1-u^2)\Delta} \tag{48}$$

$$\Phi_{00}^{(0)} = \Psi_2^{(0)} = \phi_v^2 = \frac{\theta(v)}{(1-v^2)\Delta} \tag{49}$$

$$\Phi_{02}^{(0)} = \Phi_{20}^{(0)} = 0 \tag{50}$$

It is clear from the Weyl scalars that $\tau = 1$ is a spacetime singularity and unlike the case of $0 < A < 1$ the singular hypersurface is spacelike.
The incoming components (for region II) are only $\Phi_{22}^{(0)}(u)$ and $\Psi_4^{(0)}(u)$ given by

$$\Phi_{22}^{(0)} = \Psi_4^{(0)} = \frac{\theta(u)}{(1-u^2)^2} \tag{51}$$

Similar components (for region III) are obtained for $\Phi_{00}^{(0)}$ and $\Psi_0^{(0)}$ by replacing $u \to v$. We observe that these components extent into the interaction region from their respective incoming regions in a continuous manner. However components such as $\Phi_{11}^{(0)}$ and $\Psi_2^{(0)}$ arise in the interaction region without counterparts in the incoming regions. This means there is a discontinuity (or jump) as far as these components are concerned. This type of discontinuity is classified as shock-type by CX [18] and is interpreted to originate from the gravitational shock waves, not from any current sheets or null shells. We recall in the case of the BS that the Ricci component $\Phi_{02} = a_0 b_0 \theta(u)\theta(v)$, with $a_0 = const.$ and $b_0 = const.$, also suffers from the same discontinuity. For the case of $A \neq 0$ (and $a \neq b$) we observe from (24) that there is an additional impulsive type of discontinuity originating from the occurrence of impulsive gravitational waves.

III. GEODESICS BEHAVIOUR AND TIDAL ACCELERATIONS IN REGION IV.

As was clarified in the previous section, any test particle that is imported by one of the incoming waves is forced to enter the region of interaction and arrives at the singularity in a finite interval of proper time. Now we shall study the behaviour of test particle geodesics as projected on the (τ, x, y) subspace. The first integrals of motion from the geodesics Lagrangian method are,

$$\begin{aligned}
\dot{x} &= -\frac{P_x}{\Delta^{1-A} Z^2} \\
\dot{y} &= -\frac{P_y}{\Delta^{A} Z^{-2}} \\
\dot{\tau}^2 &= \Delta^A Z^{-2} \delta_1 + \Delta^{2A-1} Z^{-4} P_x^2 + P_y^2
\end{aligned} \tag{52}$$

where δ_1 is 0 (for null) or 1 (for timelike geodesics) while P_x and P_y are constants of motion. (Δ and Z are given in equation (2)). Accelerations of the particles are obtained as,

$$\ddot{x} = \frac{P_x}{\Delta^{2-A} Z^3} \left\{ a(1+\tau)^A [A - \tau] - b(1-\tau)^A [A + \tau] \right\} \dot{\tau} \tag{53}$$

$$\ddot{y} = -\frac{A P_y}{2\Delta^{A+1}} \left\{ a^2 (1+\tau)^{2A} - b^2 (1-\tau)^{2A} \right\} \dot{\tau} \tag{54}$$

$$\begin{aligned}
\ddot{\tau} = &-\frac{P_x^2}{\Delta^{2-2A} Z^5} \left\{ a(1+\tau)^A [A - \frac{\tau}{2}] - b(1-\tau)^A [A + \frac{\tau}{2}] \right\} \\
&- \frac{A \delta_1}{2\Delta^{1-A} Z^3} \left\{ a(1+\tau)^A - b(1-\tau)^A \right\}
\end{aligned} \tag{55}$$

in which $\dot{\tau}$ is to be substituted from (52). In order to study the geodesics behaviour in the vicinity of the null singularity $\tau = 1$, we consider expansions in terms of the parameter $\varepsilon = 1 - \tau > 0$. First for the background scalar field case $A = 0$, which makes a spacelike singularity we have

$$\ddot{\tau} \sim \frac{const.}{\varepsilon^2}$$

$$\ddot{x} \sim \frac{const.}{\varepsilon^{\frac{5}{2}}}$$
$$\ddot{y} \sim 0 \tag{56}$$

where the finite terms are denoted shortly by *const.*. The fact that $\ddot{y} \sim 0$ implies that y is proportional to the affine parameter as $\varepsilon \to 1$. A simple analysis reveals also that for $A = 1$ all tidal accelerations can be made finite by the choice of suitable initial conditions. We choose for instance $P_y = 0$ within the context of the BS solution (i.e. $A = 1$ and $a = b$). This originates from the fact that the norm of the Killing vector associated with the y direction diverges. For a detailed exposition of these BS geodesics, since it is beyond our scope here we refer to elsewhere [15,19]. Our main concern here now is the scalar field in the interval $0 < A < 1$. Expansion of the above accelerations in powers of ε yield the followings

$$\ddot{\tau} \sim \frac{const.}{\varepsilon^{2-2A}} + \delta_1 \frac{const.}{\varepsilon^{1-A}}$$
$$\ddot{x} \sim \dot{\tau}\frac{const.}{\varepsilon^{2-A}}$$
$$\ddot{y} \sim \dot{\tau}\frac{const.}{\varepsilon^{A+1}}$$
$$\dot{\tau} = \sqrt{P_y^2 + c_1^2 \varepsilon^{2A-1}} \tag{57}$$

in which all *const.* terms represent finite numbers while the special constant, $c_1^2 = \frac{2P_x^2}{(a+b)^2}$. We see that as $\varepsilon \to 0$ time-like and null geodesics make no difference. The tidal accelerations are seen to be worse for $0 < A < \frac{1}{2}$ than $\frac{1}{2} < A < 1$. It turns out as a general rule that as A increases from zero toward unity the counter scalar field in the collision serves to weaken the singularity. When it reaches $A = 1$ there is no singularity remaining and $\tau = 1$ emerges as a Cauchy horizon.

It is also instructive to calculate the time of fall into the singularity as measured from the instant of collision. For this purpose we project our line element into the two dimensional space

$$ds^2 = \Delta^{1-A} Z^2 \left(\frac{d\tau^2}{\Delta} - d\theta^2 \right) \tag{58}$$

where we assumed $x = const.$,$y = const.$ and $\sigma = \cos\theta$. Now a geodesics Lagrangian treatment is equivalent to the energy integral

$$\delta_1 = \Delta^{1-A} Z^2 \left(\frac{\dot{\tau}^2}{\Delta} - \dot{\theta}^2 \right) \tag{59}$$

where δ_1 is 1(0) for timelike (null) geodesics and $\cdot$ represents derivatives with respect to the affine parameter. We obtain as a result the proper time of fall into the singularity as

$$t_0 = \int_0^1 \frac{Z^2}{\sqrt{\delta_1 \Delta^A Z^2 + \alpha^2 \Delta^{2A-1}}} d\tau \tag{60}$$

where α is a constant of integration associated with the cyclic coordinate in (58) . The time for null geodesics is obtained as,

$$t_0 = \frac{a^2}{\alpha} B_{\frac{1}{2}} \left[\frac{3}{2} - A, \frac{3}{2} + A \right] + \frac{b^2}{\alpha} B_{\frac{1}{2}} \left[\frac{3}{2} + A, \frac{3}{2} - A \right] + \frac{ab\pi}{8\alpha} \tag{61}$$

For timelike geodesics, $\delta_1 = 1$ we calculate the shortest time for test particles which is equivalent to choosing $\alpha = 0$ in equation (60). The integration gives,

$$t_0 = a B_{\frac{1}{2}} \left[1 - \frac{A}{2}, 1 + \frac{A}{2} \right] + b B_{\frac{1}{2}} \left[1 + \frac{A}{2}, 1 - \frac{A}{2} \right] \tag{62}$$

in which $B_\lambda [\mu, \nu]$ is an incomplete beta function defined in equation (38).
It is interesting to see that the time of fall is determined by the scalar charge and the incomplete beta function. We recall that the Euler beta function was encountered in the scattering problems and S-matrix of field theory. The present problem is also about scattering but instead of the complete Euler beta function we have the incomplete beta function.

IV. NULL SINGULAR CPWS AND BLACK HOLES

In this section, we shall consider another interesting horizon forming CPW solution found by CX. This solution is locally isometric to the region in between the inner and event horizons of the KN BH. The solution is described by the metric,

$$ds^2 = X \left(\frac{d\tau^2}{\Delta} - \frac{d\sigma^2}{\delta} \right) - \Delta\delta \frac{X}{Y} dy^2 - \frac{Y}{X} \left(dx - q_2 dy \right)^2 \tag{63}$$

where

$$\begin{aligned} X &= \frac{1}{\alpha^2} \left[(1 - \alpha p \tau)^2 + \alpha^2 q^2 \sigma^2 \right] \\ Y &= 1 - p^2 \tau^2 - q^2 \sigma^2 \\ q_2 &= -\frac{q\delta}{p\alpha^2} \frac{1 + \alpha^2 - 2\alpha p \tau}{1 - p^2\tau^2 - q^2\sigma^2} \end{aligned} \tag{64}$$

in which the constant parameters α, p and q must satisfy

$$\begin{aligned} 0 < \alpha \leq 1 \\ p^2 + q^2 = 1 \end{aligned} \tag{65}$$

This metric admits a CH instead of a space-like curvature singularity at $\tau = 1$. In ref[20], we have shown the relations between CPWs and the BH interiors in detail. Now let us consider a class of linearly polarized versions of the metric(63) which is isometric to the region in between the horizons of RN BH. Our aim here is to show the existence of a null singularity by employing a spherically symmetric scalar field ϕ which satisfies, the massless scalar field equation

$$\partial_\mu \left(g^{\mu\nu} \sqrt{g} \phi_\nu \right) = 0 \tag{66}$$

or equivalently

$$\left[(1 - \tau^2) \phi_\tau \right]_\tau - \left[(1 - \sigma^2) \phi_\sigma \right]_\sigma = 0 \tag{67}$$

By shifting the metric function $X \to Xe^{-\Gamma}$ we can generate an EMS solution from the known EM solution. The metric function Γ arises due to the scalar field ϕ (see ref [20] for more detail). As an EM solution we shall employ the diagonal ($q = 0$) version of metric (63) which is isometric to the RN BH. In terms of (τ, σ) the metric function Γ is obtained from the integrability conditions

$$(\tau^2 - \sigma^2)\Gamma_\tau = 2\Delta\delta\left(\tau\phi_\tau^2 + \frac{\tau\delta}{\Delta}\phi_\sigma^2 - 2\sigma\phi_\tau\phi_\sigma\right)$$

$$(\sigma^2 - \tau^2)\Gamma_\sigma = 2\Delta\delta\left(\sigma\phi_\sigma^2 + \frac{\sigma\Delta}{\delta}\phi_\tau^2 - 2\tau\phi_\tau\phi_\sigma\right) \tag{68}$$

A general class of separable solution for the scalar field ϕ is given by

$$\phi(\tau, \sigma) = \sum_n \{a_n P_n(\tau)P_n(\sigma) + b_n Q_n(\tau)Q_n(\sigma) + c_n P_n(\tau)Q_n(\sigma) + d_n P_n(\sigma)Q_n(\tau)\} \tag{69}$$

Where P and Q are the Legendre functions of the first and second kind, respectively and a_n, b_n, c_n and d_n are arbitrary constants.
a) The choice, $Q_0(\sigma) = 1, P_0(\tau) = \frac{1}{2}\ln|\frac{1+\tau}{1-\tau}|, a_n = b_n = d_n = 0$ and $c_0 = k, c_n = 0 (n \neq 0)$ is equivalent to a simple class of spherically symmetric scalar field,

$$\phi(\tau) = \frac{k}{2}\ln\left|\frac{1+\tau}{1-\tau}\right| \tag{70}$$

with $k = constant$. The new metric that represents the interaction region of linearly polarized colliding EMS fields can be written as

$$ds^2 = Xe^{-\Gamma}\left(\frac{d\tau^2}{\Delta} - \frac{d\sigma^2}{\delta}\right) - \Delta\delta\frac{X}{Y}dy^2 - \frac{Y}{X}dx^2 \tag{71}$$

where

$$e^{-\Gamma} = \left|\frac{1-\tau^2}{\tau^2 - \sigma^2}\right|^{k^2} \tag{72}$$

With the addition of this scalar field we can see from the energy momentum scalar T_α^α and $T_{\mu\nu}T^{\mu\nu}$, which are both divergent that $\tau = 1$ is a singularity. Furthermore, the fact that as $\tau \to 1$ the metric function $g^{\tau\tau} \to 0$ for the case $k^2 < 1$ implies a null singularity character. For $k^2 \geq 1$, however, it retains the spacelike character which is standard to CPW.
Using the transformations

$$t = m\alpha x, \quad y = \phi, \quad \tau = \frac{m - r}{\sqrt{m^2 - e^2}}, \quad \sigma = \cos\theta \tag{73}$$

with $m\alpha = \sqrt{m^2 - e^2}$ we obtain

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)dt^2 - e^{-\Gamma}\left(1 - \frac{2m}{r} + \frac{e^2}{r^2}\right)^{-1}dr^2$$

$$-r^2\left(e^{-\Gamma}d\theta^2 + \sin^2\theta d\phi^2\right) \tag{74}$$

Obviously this represents an extension of the RN solution with a minimally coupled scalar field without spherical symmetry. The singularity structure is investigated by calculating Ricci and Weyl scalars given in Appendix B. It is clear that coupling of scalar field destroys the BH property and the event (outer) and Cauchy (inner) horizons become spacelike singular for $k^2 > 1$. and null singular for $k^2 < 1$.

b) The choice, $b_n = c_n = d_n = 0$ equation (69) becomes

$$\phi(\tau, \sigma) = \sum_n a_n P_n(\tau) P_n(\sigma) \tag{75}$$

This class of scalar field is not spherically symmetric. Surprizingly it is regular as the CH is approached ($\tau \to 1$). For a particular case we choose $n = 2$, where the scalar field becomes,

$$\phi(\tau, \sigma) = a_1 \tau \sigma + \frac{a_2}{4}(3\tau^2 - 1)(3\sigma^2 - 1) \tag{76}$$

The metric function Γ is obtained as

$$\Gamma = a_1^2 \tau^2 + \frac{9}{4} a_2^2 \tau^2 (1 - \frac{\tau^2}{2}) - 6 a_1 a_2 \tau \sigma \Delta \delta + \\ \frac{\Delta}{4} \left\{ \frac{9}{2} a_2^2 \sigma^2 (9\tau^2 - 1) + \sigma^2 \left(4a_1^2 + 9a_2^2 - 45 a_2^2 \tau^2 \right) \right\} \tag{77}$$

which is finite as the CH is approached $\Gamma(\tau \to 1) = a_1^2 + \frac{9}{8} a_2^2$.
The Weyl and curvature scalars are also finite in the limit $\tau \to 1$. This choice of scalar field does not effect the CH in the region of interaction and therefore the spacetime remains regular. For this class of colliding EMS fields, the analytic extension of the interaction region is possible. However this analytic extension does not allow us to interpret this solution outside the ergosphere in the corresponding transformed BH spacetime. The reason simply is that, the spacetime is not asymptotically flat and the related energy of the scalar field becomes unbounded.
The solution obtained in this way can be treated as a RN BH with a scalar hair confined in between the event and Cauchy horizons.

V. DISCUSSION

It is a known fact that CPWs in general relativity result in the creation of spacelike curvature singularities and rarely in quasiregular singularities that are equivalent to CHs. CHs are important as far as the analytic extension of the resulting spacetime is concerned. In this paper, we have investigated singularities forming in the space of colliding EMS waves. This is motivated by the appearance of null singularities in BH spacetimes bombarded by pulses of scalar fields. The crucial link is the analogy between the mathematical theories of BHs and of colliding waves.
In our first example we have constructed a new class of colliding EMS waves that develops null curvature singularity in the region of interaction. In the problem under consideration, we have found that the null singularity emerges as a transition phase between a regular horizon and a spacelike singularity. Geodesics analysis and scalar curvatures reveal a systematic

weakening of divergence in the case of null singularity.
In the second example, we have used the local isometry between the CPW and BH spacetimes. This method enables us to couple scalar fields to a CPWs, where analytic exact solution is possible and then transform the resulting spacetime into BH spacetimes. With this method, we have coupled two types of scalar fields. In the first case we have used spherically symmetric scalar field and observed that the scalar field destroys the BH property and the inner and outer horizons become null singular for $k^2 < 1$ and spacelike singular for $k^2 > 1$. As a second example we couple non-spherical scalar field and observed that the inner and outer horizons remain regular. This class of solutions can be interpreted as a RN BH with a scalar hair confined in the ergosphere without a rotational symmetry.
It is important to compare our results to those of previous analyses [8,9,10]. In our case the null curvature singularities are *strong* in the sense that all tidal forces becomes unbounded. Ori has analysed the singularity inside a rotating BH using non-linear perturbation theory. His analyis concluded with a null *weak* singularity. Burko confirmed Ori's results, using numerical methods when he applied scalar field to a RN BH. We believe that the *strong* character in our case arises due to the **exact** solutions. On the other hand, Xanthopoulos [21] has shown the formation of a singularity on the null surface caused by the collision of plane gravitational and hydrodynamic waves in perfect fluids with equation of state $\epsilon = p + k, k = constant$. without a detailed analysis.
Consequently, our overall impression about the CHs is that they do not have a unique character. They may turn singular or remain regular with respect to different perturbing potentials.

VI. ACKNOWLEDGEMENT.

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VII. FIGURE CAPTIONS.

Figure 1. The spacetime diagram for colliding EMS fields. The collision occurs at point C where $u = v = 0$. In the problem considered the surface $\omega = 1 - u^2 - v^2 = 0$ or $\tau = 1$ represented by the arc AB, is null rather than spacelike.

Figure 2. The projection of region IV in (ξ, η) plane. With the transformation the null character of the arc AB in Figure 1 becomes more clear. $\xi = \eta = 1$ is the instant of collision corresponds to $u = v = 0$ or $\tau = 0$ and $\omega = 1$. However, $\xi = \eta = 0$ corresponds to $\omega = 0$ or $\tau = 1$ where the null curvature singularity occurs.

APPENDIX A: PROPERTIES OF THE EMS GEOMETRY.

The non-zero scale invariant Weyl, Ricci and curvature scalars of the collision of EMS fields are obtained as

$$
\begin{aligned}
-2\Psi_0^{(0)} &= 2A\left\{\frac{u\theta(u)}{\sqrt{1-u^2}} + \frac{\sqrt{1-u^2}\left[a(1+u)^{A-1} - b(1-u)^{A-1}\right]}{2\left[a(1+u)^A + b(1-u)^A\right]}\right\}\delta(v) \\
&\quad -\frac{u\theta(u)\theta(v)}{(1-v^2)^{\frac{3}{2}}}\left\{\frac{(2A-1)\tau}{\Delta} - \frac{\sigma}{\delta} + \frac{A\Omega}{Z}\right\} \\
&\quad +\frac{\theta(v)}{1-v^2}\left\{\frac{(2A-1)[2\tau^2(1-A)+1]}{\Delta}\right. \\
&\quad + \frac{A(A-1)}{Z}\left[a(1-3\tau)(1+\tau)^{A-1} + b(1+3\tau)(1-\tau)^{A-1}\right] \\
&\quad \left. -\frac{3A^2\Delta}{2Z^2}\Omega^2 - \frac{1}{\delta} - \frac{v}{\sqrt{1-v^2}}\left[\frac{(2A-1)\tau}{\sqrt{\Delta}} + \frac{\sigma}{\sqrt{\delta}} + \frac{A\Omega\sqrt{\Delta}}{Z}\right]\right\} \qquad (78)
\end{aligned}
$$

$$
\Psi_4^{(0)} = \Psi_0^{(0)}(u \leftrightarrow v) \qquad (79)
$$

$$
\begin{aligned}
\Psi_2^{(0)} &= \frac{\theta(u)\theta(v)}{\sqrt{1-u^2}\sqrt{1-v^2}}\left\{\frac{1-A}{\Delta} + \frac{A}{4Z^2}\left\{a^2(1+\tau)^{2A-1} + b^2(1-\tau)^{2A-1}\right.\right. \\
&\quad \left.\left. +2ab(1-2A)\Delta^{A-1}\right\}\right\} \qquad (80)
\end{aligned}
$$

$$
\begin{aligned}
4\Phi_{00}^{(0)} &= \frac{\theta(v)}{(1-v^2)\Delta Z^2}\left\{\left(1-A^2\right)\left[a^2(1+\tau)^{2A} + b^2(1-\tau)^{2A}\right]\right. \\
&\quad \left. +2ab\left(1+A^2\right)\Delta^A\right\} \qquad (81)
\end{aligned}
$$

$$
\begin{aligned}
4\Phi_{22}^{(0)} &= \frac{\theta(u)}{(1-u^2)\Delta Z^2}\left\{\left(1-A^2\right)\left[a^2(1+\tau)^{2A} + b^2(1-\tau)^{2A}\right]\right. \\
&\quad \left. +2ab\left(1+A^2\right)\Delta^A\right\} \qquad (82)
\end{aligned}
$$

$$
\Phi_{02}^{(0)} = \frac{abA^2\theta(u)\theta(v)\Delta^{A-1}}{\sqrt{1-u^2}\sqrt{1-v^2}Z^2} \qquad (83)
$$

$$
\Phi_{11}^{(0)} = \frac{(1-A^2)\theta(u)\theta(v)}{2\sqrt{1-u^2}\sqrt{1-v^2}\Delta} \qquad (84)
$$

$$
\Lambda^{(0)} = \frac{(A^2-1)\theta(u)\theta(v)}{6\sqrt{1-u^2}\sqrt{1-v^2}\Delta} \qquad (85)
$$

where $\Omega = a(1+\tau)^{(A-1)} - b(1-\tau)^{(A-1)}$.
For $A = 1$ and $a = b$, we recover the collision of Maxwell fields which is known as the BS solution. At $\tau = 1$ there exist a CH in place of a curvature singularity. For $A = 0$, we have the geometry that represents the collision of ES fields which exhibits a spacelike curvature singularity at $\tau = 1$.

For $0 < A < 1$ we have the geometry that represents the collision of EMS fields. Before the collision we have the Weyl scalars $\Psi_0^{(0)}$ and $\Psi_4^{(0)}$ in region III and II respectively. After the collision we have $\Psi_0^{(0)}, \Psi_4^{(0)}$ and $\Psi_2^{(0)}$. This indicates that part of the waves are transmitted in the region of interaction, part are reflected by each other and part of the incoming waves transforms into a Coulomb-like $(\Psi_2^{(0)})$ gravitational waves due to the non-linear interaction. Some of the Ricci scalars arise discontinuously in the interaction region. For instance, the energy momentum component $\Phi_{11}^{(0)}$ arises in this manner which has no counterpart before the instant of collision.
The non-zero energy momentum components in terms of sources are,

$$\begin{aligned}
4\pi T_{uu} &= e^{-M}\Phi_{22} = \Phi_{22}^{(0)} \\
4\pi T_{vv} &= e^{-M}\Phi_{00} = \Phi_{00}^{(0)} \\
4\pi T_{xx} &= e^{U-V}\Phi_{02} + e^{V-U}\left[\Phi_{11} - 3\Lambda\right] \\
4\pi T_{yy} &= -e^{U+V}\Phi_{02} + e^{-V-U}\left[\Phi_{11} - 3\Lambda\right]
\end{aligned} \tag{86}$$

where

$$\begin{aligned}
e^{-M} &= \frac{2\Delta^{(1-A)}Z^2}{\sqrt{1-u^2}\sqrt{1-v^2}} \\
e^{V} &= \Delta^{(\frac{1}{2}-A)}Z^2\delta^{\frac{1}{2}} \\
e^{-U} &= \sqrt{\Delta\delta}
\end{aligned}$$

in which Δ, δ and Z is given in equation (2).

APPENDIX B: THE WEYL AND MAXWELL SCALARS.

The nonzero Weyl and Maxwell scalars of the RN BH coupled with a spherically symmetric scalar field given in equation(70) are calculated and found as follows.

$$\Psi_1 = -\Psi_3 = \frac{k^2(m^2-e^2)(mr-e^2)\cos\theta\sin\theta e^{\Gamma}}{2r^3\sqrt{(r-m)^2-(m^2-e^2)}[(r-m)^2-(m^2-e^2)\cos^2\theta]}$$

$$
\begin{aligned}
\Psi_0 = \Psi_4 &= \frac{k^2(m^2-e^2)(r-m)\sin^2\theta[m(r-m)+m^2-e^2]e^{\Gamma}}{2r^3[(r-m)^2-(m^2-e^2)][(r-m)^2-(m^2-e^2)\cos^2\theta]} \\
\Psi_2 &= \frac{k^2(m^2-e^2)e^{\Gamma}}{[(r-m)^2-(m^2-e^2)]}\left\{\frac{1}{3r^2}-\frac{(mr-e^2)(r-m)\sin^2\theta}{2r^3[(r-m)^2-(m^2-e^2)\cos^2\theta]}\right\} \\
&\quad -\frac{(mr-e^2)e^{\Gamma}}{r^4} \\
\Phi_{00} = \Phi_{22} &= \frac{k^2(m^2-e^2)e^{\Gamma}}{2r^3[(r-m)^2-(m^2-e^2)]} \\
\Phi_{11} &= \frac{e^{\Gamma}}{2r^2}\left\{\frac{e^2}{r^2}-\frac{k^2(m^2-e^2)}{2[(r-m)^2-(m^2-e^2)]}\right\} \\
\Lambda &= \frac{k^2(m^2-e^2)e^{\Gamma}}{12r^2[(r-m)^2-(m^2-e^2)]}
\end{aligned}
\tag{87}
$$

where

$$
e^{\Gamma} = \left|\frac{(r-m)^2-(m^2-e^2)\cos^2\theta}{[(r-m)^2-(m^2-e^2)]}\right| \tag{88}
$$

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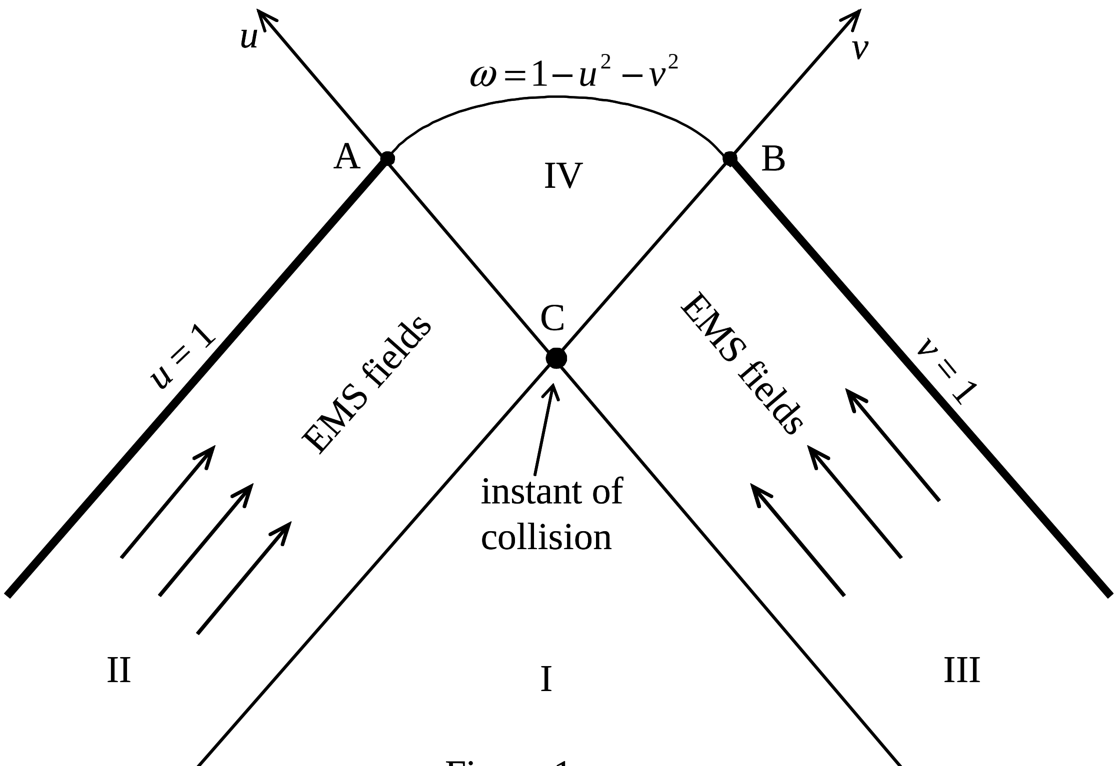
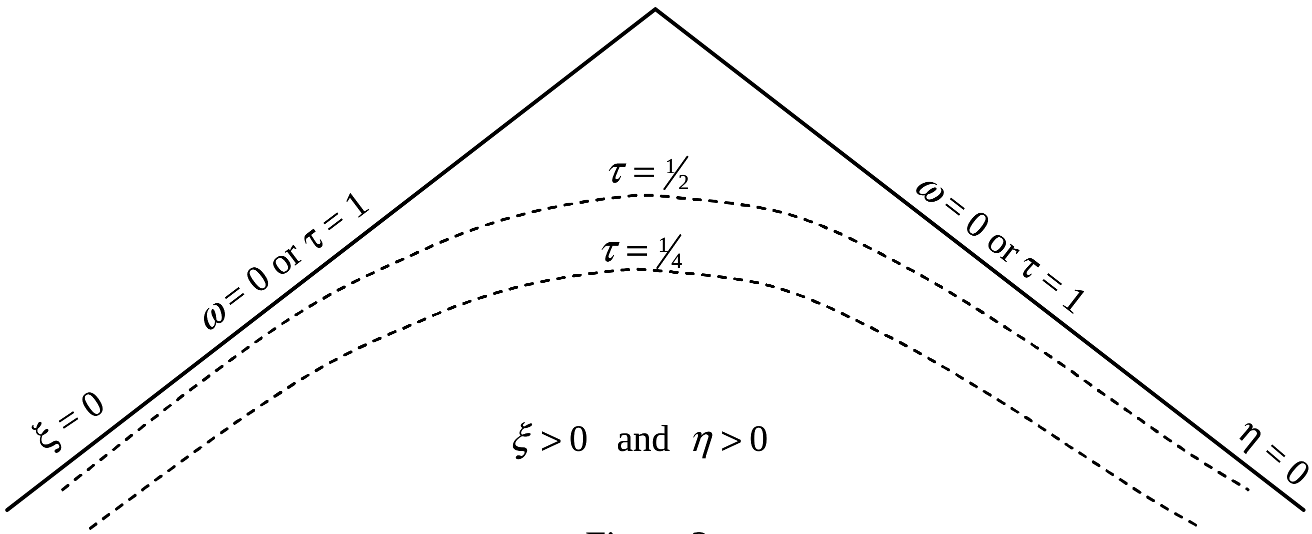

u
v
$\omega = 1 - u^2 - v^2$
A
B
IV
C
$u = 1$
EMS fields
EMS fields
$v = 1$
instant of
collision
II
I
III

Figure 1



$\omega = 0$ or $\tau = 1$
$\xi = 0$
$\tau = 1/2$
$\tau = 1/4$
$\omega = 0$ or $\tau = 1$
$\eta = 0$
$\xi > 0$ and $\eta > 0$

Figure 2